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A NEW PROOF OF PYTHAGOREAN THEOREM

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In this short paper, we introduce a new geometric proof of the Pythagorean Theorem (PT). The PT states that the sum of squares of short legs of a right triangle is equal to the square of the hypotenuse [1].

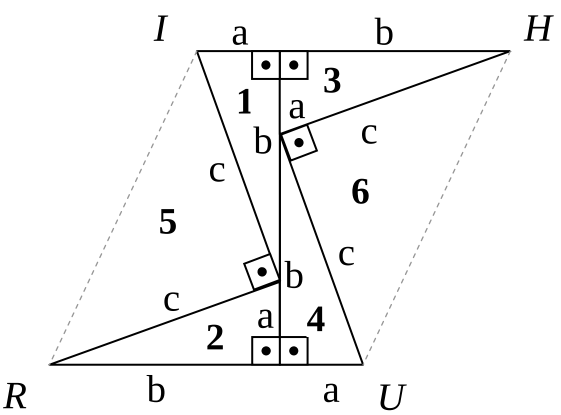
In reference to Figure 1, the area *A* of the parallelogram *RUHI* is given by:

$$A = \text{Base} \times \text{Height} = (a+b)(a+b) = a^2 + b^2 + 2ab \quad (1)$$

The same area can also be expressed as the sum of the areas of the four of right triangles 1, 2, 3 and 4 and the two isosceles right triangles 5 and 6:

$$4(ab/2) + 2(c^2/2) = c^2 + 2ab \quad (2)$$

Equating (1) and (2) we find that $c^2 = a^2 + b^2$ which completes the proof.



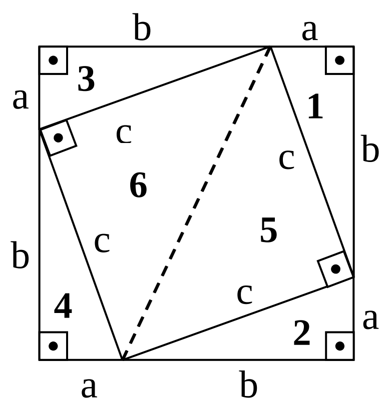


Figure 1. A parallelogram is formed by the 4 right angle triangles (numbered 1 through 4) with legs *a* and *b* and hypotenuse *c*.	**Figure 2.** By moving the triangles 1, 2, and 5 to the right, we obtain a square which gives rise to another proof of PT.

As for the relation of this proof to other proofs of PT, we see in Figure 2 that by moving triangles 1, 2, and 5 towards right, we can form a square. This square with 4 right

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triangles and 2 isosceles right triangles in it provides another proof of PT. Indeed, this is the proof suggested by the Indian mathematician Bhaskara [2]. Furthermore, by using triangles 1, 2, and 5 or 3, 4, and 6 we can obtain the proof of PT based on trapezoids which was discovered by James Garfield (1831-1881), former President of the United States [1].

To the best of our knowledge, the proof that we presented does not exist elsewhere in literature and therefore it is a new proof. However, the issue of what counts a new proof is still being debated. We reviewed many proofs of PT available on the Internet. Especially the references [3-5] provide excellent visual resources to explore different proofs of PT. Perhaps the most comprehensive resource about the proofs of PT in literature is a book named *The Pythagorean Proposition* [6] by an early 20th century mathematics professor Elisha Scott Loomis. The book is reported to have a collection of 367 proofs of the PT. Even though we could not get hold of this book, we had a chance to see many proofs that the book is reported to cover on the Internet [3]. Noting that a proof utilizing a parallelogram does not exist, we decided to publish our proof.

Regardless of the issue of whether our proof constitutes an indisputably new proof of PT, the means that we came up with proof is very interesting. The 2nd author who is teaching at Yasar University, discussed different proofs of PT at one of his freshman level discrete mathematics class meetings and asked his students to come up with a new proof of PT. (The reward was an outright A from the class.) With this great motivation, one of his students (the 1st author) cut 4 similar right triangles from cardboard and played with them over a weekend. During his search for different geometrical shapes which may give rise to a new proof of PT, the 1st author discovered the S-shaped layout of the four triangles (1 through 4, in Figure 1). When we complete the picture by drawing the dashed lines, we obtain the parallelogram which our proof is based on.

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